Instantaneous rate of change definition

Use the limit definition of the derivative to find the instantaneous rate of change. Definition and examples of instantaneous rate of change. What is an instantaneous rate of change. Limit definition of instantaneous rate of change. What does instantaneous rate of change mean. Instantaneous rate of change of displacement is a definition of.

We try to solve some examples. Both y = 2xlnÃ ¢ â¡xy = 2x ln x = y 2xlnx. What is the yyy variation rate compared to xxx when (i) x = ex = ex = and (ii) y = 4e2? Y = 4e ^ 2? Y = 4E2? The question is asking to evaluate DYDX FRAC {Text {D} y} {text {d} x} dxdyÃ ¢ to the data values of xxx and yyy. To differentiate the expression, we need to know the product rule and the differentiation of the logarithmic functions. We DYDX = DDX (2xlnÃ ¢ Â¡x) = 2DDX (XLNA Â¡x) = 2 (XA DDX (LNA Â¡x) + LNA Â¡xÃ ¢ DDX (x)) = 2 (xx + LNA Â¡ x) = 2 (1 + LNA Â¡x). begin {aligned} frac {text {d} y} {text {d}} x & = frac {text {d}} {text {d} x} left (2x ln x Right) & = 2 frac {text {d}} {text {d} x} left (x}} & = 2 left (x cdot frac {text {d}} {text {d} x} left (ln x} + ln x cdot frac {text {d}} {text {d} x} left (x right) RIGHT) & = 2 LEFT (FRAC {X} {x} + LN X RIGHT) & = 2 Left (1 + LN X RIGHT). End {} aligned dxdyÃ ¢ Ã ¢ = dxdÃ ¢ (2xlnx) = 2dxdÃ ¢ (xlnx) = 2 (xa dxdÃ ¢ (lnx) + lnxÃ ¢ dxdÃ ¢ (x)) = 2 (xxa + lnx) = 2 (1 + LNX) .Ã, (i) Now we evaluate in x = ex = ex = e. When x = ex = ex = e, dydx = 2 (1 + LNA Â¡e) = 2a (1 + 1) = 4 Frac {text {d} y} {text {d}} x = 2 (1 + ln e) = 2 cdot (1 + 1) = 4.dxdyÃ ¢ = 2 (1 + lne) = 2a (1 + 1) = 4. (ii) when y = 4e2, y = 4e ^ 2, y = 4e2, x = e2x = and ^ = 2x E2 and DYDX = 2 (1 + LNA Â¡E2) = 2a (1 + 2) = 6. frac {text {d} y} {text {d}} x = 2 (1 + ln and ^ 2) = 2 cdot (1 + 2) = 6. dxdyÃ ¢ = = 2 (1 + LNE2) = 2A (1 + 2) = 6. Un â¡_ Piazza â¡Ã ¢ A red cube has length side A, A, A, and AAA is changing with time so that A (T) = A0T2A (T) = A _ {text {0}} T ^ 2a (T) = T2 A0a. Find the instant rate of variation of the volume of the red cube as a function of time. Let the volume of the red cube is VVV. We know that V = A3 = (A0T2) 3.V = A ^ 3 = (a _ {text {0}} T ^ 2) ^ 3.v = A3 = (A0a T2) 3. You are asked to find DVDT frac {text {d} v} {text {d}} t dtdvÃ ¢. We are able to solve this problem in two ways: Solution 1: First find VVV and then DVDT Frac {text {D} V} {text {D}} t DTDVÃ ¢. We know v = a3 = (a0t2) 3 = a03a t6dvdt = 6a03Ã ¢ t5, begin {} aligned v & = a ^ 3 & = (a _ {text {0}} t ^ 2) ^ 3 & = a _ {text {0}} ^ 3 cdot t ^ 6 \\\ffrac {text {d} v} {text {d} t} & = 6a _ {text {0} } ^ 3 cdot t ^ 5, end {} aligned vdtdvÃ ¢ Ã ¢ = a3 = (a0a t2) 3 = a03a Ã ¢ t6 = 6a03Ã ¢ Ã ¢ t5, Ã ¢ and we are done! Solution 2: First we find Dadt Frac {text {d} a} {text {d}} t dtdaÃ ¢ and then dvdt frac {text {d} v} {text {d}} t dtdvÃ ¢. Using the power rule, Dadt = 2A0T. Frac {text {d} a} {text {d}} t = 2 an _ {text {0}} t.dtdaÃ ¢ = 2a0Ã ¢ t. Using the chain rule of differentiating V = A3V = A ^ = 3V A3, we have DVDT = 3A2Ã ¢ Dadt. Dfrac {text {d} v} {text {d} t} = 3a ^ 2 cdot dfrac {text {d} a} {text {d}} t = .dtdvÃ ¢ 3a2Ã ¢ dtdaÃ ¢ ¢ . After subtitude the values of 3A23A ^ 23a2 and Dadt frac {text {d} a} {text {d}} t dtdaÃ ¢, we obtain the same result above: dvt = 3a02Ã ¢ t4a 2a0t = 6a03Ã ¢ t5. Ã, â¡ â¡ dfrac {text {d} v} {text {d} t} = {3a _ text {0}} ^ 2 cdot t ^ 4 cdot 2 an _ {text {0 }} T = 6 bis _ {text {0}} ^ 3 cdot t ^ 5. _ squaredtdvÃ ¢ = 3a02Ã ¢ Ã ¢ t4a 2a0Ã ¢ t = 6a03Ã ¢ Ã ¢ t5.ã, a â¡Ã ¢ ¢ In a blue inverted cone cavity (the RRR ray vertex RRR and HHH height, water is poured into a constant LLL speed. Find the instant rate of water height variation in the cone at the moment TTT (assuming the cone is not yet completely filled). Solution to add ... to Municipality Luna Park Lifts Pilots at a height then allows them to FreeFall a certain distance before stopping safely. For example, suppose a drum drops from a height of 150 feet. Physics students can remember that the height (in of pilots, (t) seconds after free fall (and neglecting air resistance, etc.) can be carefully modeled by (T) = -16T ^ 2 + 150). Using this formula, it is easy to check that without the intervention, the runners hit the earth at (T = 2.5 sqrt {1.5} about 3.06) seconds. Suppose the travel designers decide to start slowing the fall of pilots after 2 seconds (corresponding to a height of 86 ft.). How fast will the pilots be traveling at that time? There was a position of position, but what we want calculation is a speed at a given time, ie, we want an instantaneous instantaneous instant Currently we do not know how to calculate it. However, we know from common experience how to calculate an average speed. (If we travel 60 miles in 2 hours, we know that we have had an average speed of 30 mph.) We have examined this concept in Section 1.1 we introduced the quotient of difference. We \ [\ frac {\ text {change in distance}} {\ text {changes in time}} = \ frac {\ text { "Rise ''}} {\ text {run}} = \ text {Speed media} . \] We can approximate the instantaneous speed to \ (T = 2 \) considering the average speed over a certain period of time containing \ (T = 2 \). If we realize the small time interval, we will have a good approximation. (This fact is commonly used. for example, high-speed cameras are used to track fast-moving objects. the distances are measured on a fixed number of frames to generate an accurate approximation of the speed.) consider from \ interval (T = 2 \) a \ (t = 3 \) (shortly before the riders hit the ground). at that range, the average speed is \ [\ frac {f (3) -f (2)} {3-2} = \ frac {f (3) -f (2)} {1} = -80 \ \ text {ft / s} \] where the minus sign indicates that the drivers are falling. Laughing l 'interval that we consider, we will probabilme nte get a better approximation of the instantaneous speed. On \ ([2,2,5] \) we have \ [\ frac {f (2.5) -f (2)} {2.5-2} = \ frac {f (2.5) -f (2)} {0.5} = -72 \ \ text {ft / s}. \] We can do it for smaller and smaller time intervals. For example, over a 1/10 \ (^ \ text {th} \) time interval of a second, ie, to \ ([2,2,1] \), we \ [\ frac {f (2.1) - f (2)} {2.1-2} = \ frac {f (2.1) -f (2)} {0.1} = -65.6 \ \ text {ft / s}. \] On one of a second, to \ ([2,2.01] \), the average speed is \ [\ frac {f 1/100 \ time interval (^ \ text {th} \) (2.01) -f (2) 2.01-2} {} = \ frac {f (2.01) -f (2)} {12:01} = -64.16 \ \ text {ft / s}. \] What really are the calculation is the average speed of the aggressor \ ([2.2 + h] \) for small \ values (h \). In other words, we are compucendo \ [\ frac {f (2 + h) - f (2)} {h} \] where \ (h \) is small. What we really want is for \ (h = 0 \), but this, of course, returns the familiar "\ (0/0 \) '"" ' "" indeterminate form. So we use a limit, as we did in Section 1.1. We can approximate the value of this limit with numerically small \ values (h \) as seen in Figure 2.1. It seems that the speed approached \ (- 64 \) ft / s. Compinting the limit gives directly aligns {*} \ lim_ {h \ to 0} \ frac {f (2 + h) -f (2)} {h} & â â = \ ll_ {h \ to 0 } \ frac {-16 (2 + h) ^ 2 + 150 - (-16 (2) ^ 2 + 150)} {h}}}} lim_ {h \ to 0} \ frac {-64h-16h ^ 2 {h}} \\ & = \ lim_ {h \ to 0} -64 -16h \\\ & = - 64. \ end {align *} \] Figure 2.1: approximation of instantaneous speed with average speed over a short period time \ (h \). Graphically, we can see the speed averages that we calculated numerically as the slopes of Secant lines on \ graph (f \) passing through the points \ ((2, f (2)) \) and \ ((2 + h , f (2 + h)) \). In Figure 2.2, the Secant line corresponding to \ (h = 1 \) is shown in three contexts. Figure 2.2 (A) shows a "enlarged" version of \ (f \) with suolinea Secant. In (b), we magnify around the points of intersection between \ (f \) and Secant line. NOTICE AS WELL THIS LINE SECENTANTE APPROSTINARE \ (F \) between these two points - is a common practice to approximate functions with straight lines. As \ (h \ to 0 \), these secant lines approach the tangent line, a line passing through the point \ ((2, f (2)) \) with the special slope of \ (- 64 \). In parts (c) and (d) of Figure 2.2, we magnify the point \ ((2,86) \). In (c) we see the Secant line, which approximation \ (f \), but not so well the tangent line shown in (D). 2.2: Parts (a), (b) and (c) shows the secant line a (x) text {with} h = 1), enlarged in different quantities. Part (D) shows the tangent line A (f text {at} x = 2.) We have just introduced a series of important concepts that I will stow out more within this section. First of all, we are formally defined. Definition 7: derivative on a point is a continuous function on an open interval (I) and leave (C). The derivative of (f) a (c), denoted (f ^ first (c), is [{h {h the limit exists. If the limit exists, we say that \ (f \) is differentiable \ (c \)}; If the limit does not exist, \ (f \) is not differentiable at \ (c \)}. If \ (f \) is differentiable at every point \ (I \), then \ (f \) is differentiable on \ (I \). Definition 8: Line Tangent Let \ (f \) be continuous on an open interval \ (I \) and differentiable to \ (c \), for some \ (c \) in \ (I \). The with equation line \ (\ ell (x) = f ^ \ Prime (c) (x-c) + f (c) \) is the tangent line to the \ graph (f \) a \ (c \); That is, is the line through \ ((c, f (c)) \) whose slope is the derivative of \ (f \) a \ (c \). Some examples will help us to understand these definitions. Example 32: search of derivatives and tangent lines â â Let \ (f (x) = 3x ^ 2 + 5x-7 \). Find: \ (f ^ \ Prime (1) \) The equation of the tangent line to the \ graph (f \) a \ (x = 1 \). \ (F ^ \ Prime (3) \) The equation of the tangent line to the graph \ (f \) a \ (x = 3 \). Solution calculate directly using the definition 7. \ [\ begin {align *} f ^ \ Prime (1) & = \ LIM_ {H \ to 0} \ frac {f (1 + h) -f (1)} { h} \\ & = \ lim_ {h \ to 0} \ frac {3 (1 + h) ^ 2 + 5 (1 + h) -7 - (3 (1) ^ 2 + 5 (1) -7) } {h} \\ & = \ lim_ {h \ to 0} \ frac {^ 2 + 3h 11h} {h}}} {h \ = \ lim_ {h \ to 0} 3h + 11 = 11. \ end {align *} \] the tangent line on \ (x = 1 \) has a slope of \ (f ^ \ prime (1) \) and passes through the point \ ((1, f (1)) = (1, 1) \). Therefore the tangent line has the equation, in the form of point-slope, \ (y = 11 (x-1) + 1 \). In the form of interception of the slope we \ (y = 11x-10 \). Again, using the definition, \ [\ begin {align *} f ^ \ prime (3) & = \ lim_ {h \ to 0} \ frac {f (3 + h) -f (3)} {h} \ \ = \ lim_ {h \ to 0} \ frac {3 (3 + h) ^ 2 + 5 (3 + h) -7 - (3 (3) ^ 2 + 5 (3) -7)} { h} \\ & = \ lim_ {h \ to 0} \ frac {^ 2 + 3h 23h} {h} \\ & = \ lim_ {h \ to 0} 3h + 23 = 23. & \\ \ end { aligns *} \] the tangent line to \ (x = 3 \) has a slope of \ (23 \) and passes through the point \ ((3, f (3)) = (3.35) \). Therefore the tangent line has the equation \ (y = 23 (x-3) +35 = 23x-34 \). A graph of \ (f (x) = 3x ^ 2 + 5x-7 \) and its tangent lines to \ (x = 1 \) and \ (x = 3 \). Figure 2.3: A graph of \ (f (x) = 3x ^ 2 + 5x-7 \) and its tangent lines to \ (x = 1 \) and \ (x = 3 \). Another important line that can be created using the information is derived from the normal line. It is perpendicular to the tangent line, then its slope is the reverse - the reciprocal of the slope of the tangent line. Definition 9: normal line Let \ (f \) be continuous on an open interval \ (I \) and differentiable to \ (c \), for some \ (c \) in \ (I \). The normal line to the graph of \ (f \) a \ (c \) is the line with equation \ [n (x) = \ frac {-1} {f ^ \ prime (c)} (xc) + f (c) \] where \ (f ^ \ Prime (c) EQ 0 \). When \ (f ^ \ Prime (c) = 0 \), the normal line is the vertical line through \ (\ Big (C, F (c) \ Big) \); ie, \ (x = c \). Example 33: Search of lines normal equations Let \ (f (x) = 3x ^ 2 + 5x-7 \), as in Example 32. Find the equations of the normal lines to the \ graph (f \) a \ (x = 1 \) and \ (x = 3 \). Figure 2.4: a \ graph (f (x) = 3x ^ 2 + 5x-7 \), along with its normal line to \ (x = 1 \). Solution In the example 32, we found that \ (f ^ \ Prime (1) = 11 \). So to \ (x = 1 \), the normal line will have a slope \ (- 1/11 \). An equation for the normal line is \ [n (x) = \ frac {-1} {11} (-1} {11} (X-1) +1. \] The normal line is drawn with \ (y = f (x) \) in Figure 2.4. Note how the line looks perpendicular to \ (f \). (a key word here is "look." "Mathematically, we say that the normal line is perpendicular to \ (f \) a \ (x = 1 \) as the slope of the normal line is the opposite - reciprocal of the slope of the tangent line. However, the normal lines might not seem always perpendicular. the aspect ratio of the image of the graph An important role in this.) We also discovered that (f ^ first (3) = 23), then the normal line to the graph of (x = 3) will be sloped (- 1/23). An equation for the normal line is [n (x) = frac {-1} {23} (x-3) +35. The linear functions are easy to work with; Many functions that arise during the resolution of real problems are not easy to work with. A common practice in solving mathematical problems is to approximate difficult functions with no, so difficult functions. The lines are a common choice. It turns out that at any given point on the graph of a differentiable function (f), The best linear approximation A (F) is its tangent line. This is one of the reasons to spend a considerable time to find tangent lines to the functions. A type of function that does not benefit from a tangent approximation is a line; It is quite simple to recognize that the tangent line to a line is the line itself. Let's look at this in the following example. Example 34: find the derivative of a line considered (x) = 3x + 5). Find the tangent line equation to (x = 1) and (x = 7). Solution We find the slope of the tangent line using the definition 7. [Begin {Align *} f ^ Prime (1) & = LIM_ {h})} {H}}}}} & = Lim_ {H to 0} frac {3 (1 + h) +5 - (3 + 5)} {h}}}} Lim_ {h brac {3h} {h} & = lim_ {h} 3 & = 3. end {align *}] We have just found that). Ie, we found the instantaneous rate of the change of (x) = 3x + 5) is (3). This is not surprising; The lines are characterized by being the only functions with a constant rate of change. That change rate is called the slope of the line. Because their rates of change are constant, their instant change rates are always the same; I am all the slope. Then given a line (f (x) = ax + b), the derivative at any point (x) will be; ie, (f ^ first (x) = a). Now it's easy to see that the tangent line to the graph of (f) a (x = 1) is only (f), with the same true for (x = 7). We often wish to find the tangent line to the graph of a function without knowing the true derivative of the function. In these cases, the best we could be able to do is approximate the tangent line. They show him in the next example. Example 35: numerical approximation of the tangent line approximately the tangent line equation to the chart of (x) = sin x) a (x = 0). Figure 2.5: (f (x) = x_) graphics with an approximation to its tangent line to (x = 0). Solution To find the tangent line equation, we need a slope and a point. The point is given to us: (0, sin 0) = (0.0). To calculate the slope, we need the derivative. This is where we will do an approximation. Recall that [f ^ Prime (0) about the frac {sin (0 + h) - sin 0} {h} for a small value of (h). We choose (somewhat arbitrarily) to leave (H = 0.1). Then [F ^ Prime (0) approx. FRAC {sin (0.1) - sin 0} {0.1} sin 0} {0.1} 50 0.9983. Then our approximation of the tangent line equation is (Y = 0.9983 (X-0) +0 = 0.9983x); It is scratched in figure 2.5. The graph seems to imply the approximation is pretty good. RECALLS FROM SECTION 1.3 THAT (LIM_ {x 0} FRAC {sin x} x = 1 sin x} x = 1), Meaning for values Night 0, (Sin X X). Since the slope of the line (y = x) is 1 at (x = 0), it should seem reasonable that "the slope of (x) = sin x) '' is Near 1 to (x = 0). In fact, since we approached the value of the slope to be (0.9983), we could guess the real value is 1. We will return to this next. Consider example Example 32. For Find the derivative of (x = 1), we needed to evaluate a limit. To find the derivative of (x = 3), we need to evaluate a Limit. We have this process: this process describes a function; given an input (the value of (C), we return exactly one output (the value of (f ^ first (c)). The box "Do something" is where boring work occurs (taking the limits) of this function. Instead of applying this function repeatedly for different values (c), only to apply each time to the variable (x). So we take a just once. The process now seems: the output is the "derivative function", "(f ^ first (x)). The function (f ^ first (x) will take a number (c) As input and return the derivative of (F). This requires a definition. Definition 10: Derivative function leaves (f) is a differentiable function on an open interval (I). The function [F ^ (x) = lim_ {h} frac {f (x + h) -f (x)} {h} is the derivative of (f) . Notation: leaves (y = f (x)). The following notions represent all the [Prime (x) = y ^ first = frac {dy} {dx} f) = frac {d} {dx} (y). Important: the notation (frac {} {DY DX}) is a symbol; It is not the fraction "(DY / DX) '' the notation, while a bit of confusion at the beginning, it was carefully chosen a fraction - .. looking symbol was chosen because the derivative has many fraction similar properties among others. Places, we see these properties at work when talking about the derivative unit, when we discuss the chain rule, and when we learn of integration (arguments that appear in the following sections and chapters). Examples will help us To understand this definition. Example 36: find the derivative of a function is (f (x) = 3x ^ 2 + 5x-7) as in Example 32. Find (F ^ Prime (X))} . Solution: We apply Definition 10. [Begin {Align} * F ^ Prime (x) = & Lim_ {h \ t 0} frac {f (x + h) -f (x)} {h} & = lim_ {h 0} frac {3 (x + h) ^ 2 + 5 (x + h) -7- (3x ^ 2 + 5x-7)} {h} & = lim_ {h 0} frac {3h ^ 2 + 6xh + 5h} {h} & = lim_ {h} 3h + 6x + 5 & = 6x + 5 END {ALIGN *} So (f ^ first (x) = 6x + 5). Recall previously discovered that (f ^ first (1) = 11) and (f ^ first (3) = 23). Note our new calculation of (f ^ first (x) say these facts. Example 37: Calculation of the derivative of a function is (f (x) = frac {1} {x + 1}). Find (f ^ first (x)). Solution: We apply Definition 10. [F ^ Prime (x) = Lim_ {h 0} frac {f (x + h) -f (x)} {h} = {h lim_ 0} frac {frac {1} {x + h + 1} - frac {1} {x + 1}} {h}] Now find common denominator then subtract; Traction (1 / h) front to facilitate reading. [Begin {align *} & = lim_ frac {1} {h} {h} cdot {x + 1} {(x + 1) (x + h + 1)} - frac {x + h + 1} {(x + 1) (x + h + 1)} right) & = lim_ {h 0} frac 1h cdot left (frac {x + 1- (x + h + 1)} {(x + 1) (x + h + 1)} right) & = lim_ {h 0} frac1h cdot left (frac {-h} {(x + 1) (x + h + 1)} right) & = lim_ {h} frac {-1} {(x + 1) (x + h + 1)} & = frac {-1} {(x + 1) (x + 1)} & = frac {-1} {(x + 1) ^ 2} end {align *} So (f ^ first (x) = frac {-1} {(x + 1) ^ 2}). To practice using our notation, we could also have been [frac {d} {dx} left (frac {1} {x + 1} right) = frac {-1} {(x + 1) ^ 2} Example 38:. Finding the derivative of a function finds the derivative of (x) = sin x)} Solution Before applying Definition 10, note that once this is found, we can find the real one. Tangent line A (F (x) = sin x) a (x = 0), while we opted for an approximation in Example 35. We found that when (f (x) = sin x), (f ^ first (x) = so x). This should be a bit surprising; The result of a boring limit process and the sinus function is a pleasant function. Then again, maybe this is not entirely amazing. The sinus function is periodic - repeats at regular intervals. Therefore its exchange rate also repeats on the same regular intervals. We had to be derivative would be periodic; Now we know exactly what periodic function is. Retenging for example 35, we can find the slope of the rectum tangent a (x) = sin x) a (x = 0) using our derivative. We approximated the slope as (0.9983); Now we know the slope is exactly (so 0 = 1). Example 39: Finding the derivative of a set-up function defined the derivative of the absolute value function, [F (X) = | X | = Left {begin {array} {} cc -x and x