



Differentiation of division of two functions

What if I told you that there was a really easy way to remember the quotient rule for derivatives? Jenn, Founder Calcworkshop®, 15+ Years Experience (Licensed & Certified Teacher) Anything to make calculus easier right? Well then, let's go! What Is The Quotient Rule The quotient rule is a method for differentiating problems where one function is divided by another. The premise is as follows: If two differentiable functions, f(x) and g(x), exist, then their quotient is also differentiable (i.e., the derivative of the quotient Rule Formula Fun Mnemonic - Hi Dee Ho Now, I have to admit that this formula looks a bit intimidating at first, but there's an easy pattern to help us remember it and a fun saying that makes memorizing the steps super easy! The trick is to always start with the bottom function and end with the bottom function squared. I like to call this function "Ho" because it's "low," and that means the top function is called "Hi" because it's "high up." So, the saying goes something like this — Ho-Dee-Hi minus Hi-Dee-Ho over Ho-Ho! Quotient rule is formally read as the bottom times the derivative of the top, minus the top times the derivative of the function, as the quotient rule is formally read as the bottom times the derivative of the top. the bottom, all divided by the bottom squared. But let's be honest, it's much more fun to say "HoHo" than "bottom squared," am I right? Quotient Rule Examples As always, it's best to see how this formula works in practice, so let's look at an example where we will use the quotient rule to differentiate the function below. Ex) Derivative Of A Fraction Quotient Rule Derivative Ex) Common Mistake And just as we saw with the product rule, I want to caution you and help you to avoid a common mistake. The derivatives, as the example below nicely demonstrates. The Derivative Quotient Does NOT Equal The Quotient Derivative What is so interesting about this derivative rule is how closely it relates to our understanding of the product rule, except for a minus instead of a plus. But there is a small warning. While the quotient rule is very similar to the product rule. So please take your time and check your signs! Ex) Instantaneous Rate Of Change Let's look at another example so you can get more comfortable with this new differentiation rule. Evaluate The Derivative Of The Function At The Given Point So, we just found the instantaneous rate of change for f(x) when x = 1, all thanks to our new rule! Cool, huh? Together we will work through several examples in detail, color coding, and chanting our fun Ho-dee-Hi jingle along the way, so you will quickly master this fantastic rule. And we will look at how to quotient rule helps us find the rate of change, or instantaneous velocity, of one function divided by another. Let's get to it! Video Tutorial w/ Full Lesson & Detailed Examples (Video) Get access to all the courses and over 150 HD videos with your subscription Monthly, Half-Yearly, and Yearly Plans Available Get My Subscription Now Not yet ready to subscription for a spin with our FREE limits course In calculus, the quotient rule is used to find the derivative of a function which can be expressed as a ratio of two differentiable functions. In other words, the quotient rule allows us to differentiate functions: $f(x) = x^2$ and g(x) = x Now say we wanted to find the derivative of One approach to find the derivative would be to simplify the function and then differentiate it. So the derivative of the fraction f(x)/g(x) is just 1. Now, out of interest, let's calculate f'(x)/g'(x) = 2x The derivative of a quotient is not equal to the quotient of derivatives. To differentiate a quotient, let's calculate f'(x)/g'(x) = 2x The derivative of a quotient is not equal to the quotient of derivatives. To differentiate a quotient, let's calculate f'(x)/g'(x) = 2x The derivative of a quotient is not equal to the quotient of derivatives. To differentiate a quotient, let's calculate f'(x)/g'(x) = 2x The derivative of a quotient is not equal to the quotient of derivatives. To differentiate a quotient, let's calculate f'(x)/g'(x) = 2x The derivative of a quotient is not equal to the quotient of derivatives. you cannot just take the derivative of the numerator and divide it by the derivative of a quotient rule to find the derivative of a quotient rule to find the derivative of a product). The Quotient Rule Formula For Differentiation If two functions f(x) and g(x) are differentiable (i.e. the derivatives of f(x) and g(x) exist), then their quotient (f(x)/g(x)) is differentiable, and the derivative can be found as follows: The formula for the quotient rule in words The Quotient Rule states that the derivative of a quotient is equal to the denominator times the derivative of the numerator minus the numerator times the derivative of the denominator, all divided by the square of the denominator. Two tricks to remember the quotient rule Similar to the product rule, you need to find the derivative of f(x) and then find the derivative of g(x) and there is also a division which needs to be performed. This makes the quotient rule a little more difficult to memorize compared to the product rule: 1. You always start with the bottom function (denominator) and end with the bottom function squared. And if that doesn't work... 2. Here is a jingle which you can use: Lo d hi minus hi d low, all over the square of what's below. Here the lo, refers to the denominator, hi refers to the denominator, hi refers to the numerator and d refers to "the derivative of". It's really silly, but that's why it works! When to use the quotient rule In calculus, the quotient rule can be applied when the function you want to differentiate consists of a quotient (or fraction), and the numerator and denominator of the quotient are both differentiable - it's derivatives because: The numerator (x) is differentiable - it's derivative is 1 The denominator (x2) is differentiable - it's derivative is x (Of course you could also first simplify the function to 1/x and then differentiate it and get the same result) Examples Using The Quotient rule works is by looking at some examples. Using the quotient rule For the first example, let's find the derivative of Let's call the numerator f(x), so f(x) = x2 + 1Let's call the denominator g(x), so g(x) = x3 This means that: f'(x) = 2xg'(x) = 3x2 We can now apply the quotient rule to find the derivative of f(x)/g(x). The formula And then we just simplify using factorization and the rules of exponents to find the final answer Using the quotient rule to find the derivative of f(x)/g(x). The formula for the quotient rule is Now we can plug f(x), f'(x), g(x) and g'(x) into the formula And then we just simplify using factorization and the rules of exponents to find the final answer Using the quotient rule is Now we can plug f(x). rule with trig functions As another example, we can use the quotient rule to find the derivative of tan(x) First recall that tan(x) can be expressed as sin(x) divided by cos(x) Let's call the numerator f(x), so g(x) = -sin(x) We can now apply the quotient rule to find the derivative of tan(x) can be expressed as sin(x) divided by cos(x) Let's call the numerator f(x), so g(x) = -sin(x) We can now apply the quotient rule to find the derivative of tan(x) can be expressed as sin(x) divided by cos(x) Let's call the numerator f(x), so g(x) = -sin(x) We can now apply the quotient rule to find the derivative of tan(x) can be expressed as sin(x) divided by cos(x) and tan(x) can be expressed as sin(x) divided by cos(x). the derivative of f(x)/g(x). The formula for the quotient rule is Now we can plug f(x), f'(x), g(x) and g'(x) into the formula: Then simplify remembering the trig identity sin2(x) + cos2(x) = 1 and recalling that 1/cos(x) is equal to sec(x). Using the quotient rule, the derivative of tan(x) is equal to sec(x). Product and Quotient Rule Proof of the Quotient Rule There are a number of ways to prove the quotient rule. Here we will look at proving the quotient rule using: First principles - the derivative definition and the product ruleThe product and chain rulesNatural logarithms and the chain rule are a number of ways to prove the quotient rule. Here we will look at proving the quotient rule using: First principles - the derivative definition and the product ruleThe product and chain rulesNatural logarithms and the chain rulesNatural logarithms are a number of ways to prove the quotient rule using first principles Let F(x) = f(x)/g(x) The definition for the derivative of F(x) is Then we put F(x+h) and F(x) in This can be rewritten by finding the lowest common denominator in the numerator (g(x+h).g(x)) and taking the 1/h out. The next step is needed to make life easier a little later on. What we do is we add 0 to the numerator, which does not change the value at all. But instead of adding a straight 0, we add it in the form -f(x)g(x) + f(x)g(x) Now we swap the two denominators around - we are allowed to do this since we are multiplying fractions. Next we use one of the properties of limits which says that the limit of a product of two functions is equal to the product of their limits. We then also use the fact that the limit as h approaches zero of g(x+h) is simply g(x) Now we split the large fraction up, and take out a common g(x) from the first part and a common -f(x) from the second part. Another property of limits is that the limit of each part of the products Now you may notice that two of the above limits look familiar - they are the exact definitions for f'(x) and g'(x). Lastly, the limit as h approaches 0 of g(x) and f(x) remain g(x) and f(x) quotient rule. Proving the quotient rule using implicit differentiation and the product rule Let y = f(x)/g(x), then we want to find y'. First let's rearrange the equation: Now we can take the derivative on the left and right hand sides of the equation (by using the product rule on the right hand side) Next, substitute y = f(x)/g(x) into the above equation (by using the product rule on the right hand sides of the equation (by using the product rule on the right hand side) Next, substitute y = f(x)/g(x) into the above equation (by using the product rule on the right hand side) Next, substitute y = f(x)/g(x) into the above equation (by using the product rule on the right hand side) Next, substitute y = f(x)/g(x) into the above equation (by using the product rule on the right hand side) Next, substitute y = f(x)/g(x) into the above equation (by using the product rule on the right hand side) Next, substitute y = f(x)/g(x) into the above equation (by using the product rule on the right hand side) Next, substitute y = f(x)/g(x) into the above equation (by using the product rule on the right hand side) Next, substitute y = f(x)/g(x) into the above equation (by using the product rule on the right hand side) Next, substitute y = f(x)/g(x) into the above equation (by using the product rule on the right hand side) Next, substitute y = f(x)/g(x) into the above equation (by using the product rule on the right hand side) Next, substitute y = f(x)/g(x) into the above equation (by using the product rule on the right hand side) Next, substitute y = f(x)/g(x) into the above equation (by using the product rule on the right hand side) Next, substitute y = f(x)/g(x) into the above equation (by using the product rule on the right hand side) Next, substitute y = f(x)/g(x) into the above equation (by using the product rule on the right hand side) Next, substitute y = f(x)/g(x) into the above equation (by using the product rule on the right hand side) Next, substitute y = f(x)/g(x) into the above equation (by using the product rule on ultimately want to find the derivative of y (y') so now we solve for y' and find the formula for the quotient rule. Proving the quotient as a product We can find F'(x) by using the product rule To find (g(x)-1)' we can apply the chain rule. The chain rule says we first take the derivative of g(x)-1 in terms of g(x) (= (-1)g(x)-2) and then multiply that by the derivative of g(x) in terms of x (= g'(x)). Next we bring the negative exponents to the denominator and then simplify by finding the greatest common denominator. We are left with the formula for the quotient rule. Proving the quotient rule using ln and the chain rule Let y = f(x)/g(x) To start, take the natural logarithm (ln) on both sides Next apply the properties of logs to split the right hand side's quotient to a difference of logs Now take the derivative of ln(x) with respect to x is 1/x. In a similar way, the derivative of ln(y) with respect to y is 1/y, the derivative of ln(f(x)) with respect to f(x) is 1/f(x) and the derivative of ln(g(x)) with respect to g(x) is 1/g(x). To find the derivatives of each of the terms with respect to the inner function of ln, and then multiplying that by the derivative of the inner function. Lastly, isolate y', substitute y = f(x)/g(x) in, and then simplify to solve for y' The quotient rule in terms of u and v With regards to notation, the quotient rule is sometimes easier to express if you use u and v to represent the numerator and denominator respectively. This can be more compact than using the function notation f(x) and g(x). Using the variable u for the numerator, and v for the denominator, the quotient rule for finding the derivative of the function u/v can be expressed as: DerivativeIt lets you quickly look up derivatives of trigonometric, exponential and natural logarithmic functions Learning Objectives State the constant, constant multiple, and power rules. Apply the sum and difference rules to combine derivatives. Use the product rule for finding the derivative of a product of functions. Extend the power rule to functions with negative exponents. Combine the differentiation rules to find the derivative of a polynomial or rational function. Finding derivatives of functions by using the definition of the derivative can be a lengthy and, for certain functions, a rather challenging process. For example, previously we found that \[\dfrac{d}{dx}\left(\sqrt{x}} onumber\] by using a process that involved multiplying an expression by a conjugate prior to evaluating a limit. The process that we could use to evaluate \(\dfrac{d}{dx}\left(\sqrt[3]{x}\right)\) using the definition, while similar, is more complicated. In this section, we develop rules for finding derivatives that allow us to bypass this process. We begin with the basics. The functions \(f(x)=c\) and \(g(x)=x^n\) where \(n\) is a positive integer are the building blocks from which all polynomials and rational functions are constructed. To find derivative to find the derivative to find the derivative of the constant function, (f(x)=c). For this function, both (f(x)=c) and (f(x+h)=c), so we obtain the following result: $[h\to 0] drac \{c-c\} \{h\} / [4pt] \&= \lim_{h\to 0} drac \{c-c\} \{h\} / [4pt] \&= \lim_{h\to 0} drac \{0\} \{h\} / [4pt] \&= \lim_{h\to 0} drac \{0\} \{h\} / [4pt] \&= \lim_{h\to 0} drac \{c-c\} \{h\} / [4pt] \&= \lim_{h\to 0} drac \{0\} \{h\} / [4pt] \&= \lim_{h\to 0} drac \{c-c\} (h\} / [4pt] \&= \lim_{h\to 0} drac \{a-c\} / [4pt] \& (a-c) / [4pt] \& (a-c) / [4pt] \& (a-c) / [4pt$ is called the constant rule. It states that the derivative of a constant function is zero; that is, since a constant function is \(0\). We restate this rule in the following theorem. The Constant function is \(0\). We restate this rule in the following theorem. rule as $(\frac{d}{dx}(c)=0.)$ Example $(\frac{d}{dx}(c)=0.)$ Exercise $(\frac{1})$ Find the derivative of (g(x)=-3). Hint Use the preceding example as a guide Answer 0 We have shown that $(\frac{d}{dx}(c)=0.)$ Exercise $(\frac{d}{dx}(c)=0.)$ Exercise $(\frac{d}{dx}(c)=0.)$ Exercise $(\frac{d}{dx}(c)=0.)$ Example ($\frac{d}{dx}(c)=0.$) Exercise $(\frac{d}{dx}(c)=0.)$ E $dx = \frac{1}{2}x^{-1/2}$. on umber At this point, you might see a pattern beginning to develop for derivatives of the form $(dfrac{d}{dx})$. We continue our examination of derivative formulas by differentiating power functions of the form $(f(x)=x^n)$. where \(n\) is a positive integer. We develop formulas for derivatives of this type of function in stages, beginning with positive integer powers. Before stating and proving the general rule for derivatives of functions of this form, we take a look at a specific case, \(\dfrac{d}{dx}(x^3)\). As we go through this derivation, pay special attention to the portion of the expression in boldface, as the technique used in this case is essentially the same as the technique used to prove the general case. Example $(\frac{d}{dx}) = \frac{1}{dx} + \frac{1}{d$ =\lim_{h \to 0}\dfrac{x^3+3x^2h+3xh^2+h^3-x^3}{h}\) Notice that the first term in the expansion of \((x+h)^3\) is \(x^3\) and the second term is \(3x^2h\). All other terms contain powers of \(h\) that are two or greater \(\\displaystyle = \\lim_{h \to 0} \\dfrac{3x^2h+3xh^2+h^3}{h}\) In this step the \(x^3\) terms have been cancelled, leaving only terms containing \(h\). \(\displaystyle =\lim_{h \to 0}\dfrac{h(3x^2+3xh+h^2)}{h}\) Factor out the common factor of \(h\). \(\displaystyle =\lim_{h \to 0}(3x^2+3xh+h^2)) After cancelling the common factor of \(h\), the only term not containing \(h\) is \(3x^2\). \(=3x^2\) Let \(h\) go to \(0\). Exercise \(\PageIndex{2}\) Find \(\dfrac{d} $dx = \frac{dx}{dx} =$ conclusions from specific examples, we note that when we differentiate $(f(x)=x^3)$, the power on (x) in the derivative decreases by 1. The following theorem states that the power rule holds for all positive integer powers of (x). We will eventually extend this result to negative integer powers. Later, we will see that this rule may also be extended first to rational powers of \(x\) and then to arbitrary powers of \(x\). Be aware, however, that this rule does not apply to functions in which a constant is raised to a variable power, such as \(f(x)=3^x\). The Power Rule Let \(n\) be a positive integer. If \(f(x)=x^n\), then \[f' $(x)=nx^{n-1}.) Alternatively, we may express this rule as [\dfrac{d}{dx}\left(x^n\right)=nx^{n-1}.] Proof For (f(x)=x^n) where (n) is a positive integer, we have [f(x)=\lim_{h\to 0}] dfrac{(x+h)^n-x^n}{h}. on umber [Since ((x+h)^n-x^n){h}. on umber] Since ((x+$ see that \((x+h)^n-x^n=nx^{n-1}h+binom{n}{2}x^{n-2}h^2+binom{n}{3}x^{n-3}h^3+...+nxh^{n-1}+h^n.\) Next, divide both sides by h: \(\dfrac{(x+h)^n-x^n}{h}=\sqrt{n-1}+h^n.\) Next, divide both sides by h: \(\dfrac{(x+h)^n-x^n}{h}=\sqrt{n-1}+h^n.\) Next, divide both sides by h: \(\dfrac{(x+h)^n-x^n}{h}=\sqrt{n-1}+h^n.\) Next, divide both sides by h: \(\dfrac{x^{n-1}+h^n}{h}=\sqrt{n-1}+h^n.\) Next, divide both sides by h: \(\(\dfrac{x^{n-1}+h^n}{h}=\sqrt{n-1}+h^n.\) Next, dint side both sides by h: \(\(\dfrac{x^{n-1}+h^n}{h}=\sqrt{n-1} ${2x^{n-2}h+binom{n}{3}x^{n-2}h+binom{n}{3}x^{n-2}h+binom{n}{3}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^{n-2}h+binom{n}{2}x^$ rule. Solution Using the power rule with (n=10), we obtain $(f(x)=10x^{10-1}=10x^{9})$. Hint Use the power rule with (n=7.) Answer $(f(x)=7x^{6})$ We find our next differentiation rules by looking at derivatives of sums, differences, and constant multiples of functions. Just as when we work with functions, there are rules that make it easier to find derivatives of functions that we add, subtract, or multiply by a constant. These rules are summarized in the following theorem. Sum, Difference, and Constant Multiple Rules Let \(f(x)\) and \(g(x)\) be differentiable functions and \(k\) be a constant. Then each of the following equations holds. Sum Rule. The derivative of the sum of a function (g) is the same as the sum of the derivative of (g). $[dfrac{d}{dx}\big(f(x)+g(x)\big)+dfrac{d}{dx}\big(f(x)+g(x)\big)]$ that is, $[text{for }s(x)=f(x)+g(x),quad s'(x)=f'(x)+g'(x)$. onumber Difference Rule. The derivative of the difference of a function (g) is the same as the difference of the derivative of (g): $[dfrac{d}{dx}(f(x)-g(x))]$ that is, $[dfrac{d}{dx}$ multiplied by a function (f) is the same as the constant multiplied by the derivative: $[(dfrac{d}{dx}))$ and (g(x)), we set (f(x)) and (g(x)) and (g(x)) and (g(x)), we set (f(x)) and (g(x)) are (g(x)) and ($(x) = \lim_{h \to 0} \det(x+h) - f(x) + \frac{h \to 0} \det(x+h) - g(x) + \frac{h \to 0} \det(x$ derivative of $(g(x)=3x^2)$ and compare it to the derivative of $(f(x)=x^2)$ Solution We use the power rule directly: $[g'(x)=dfrac{d}{dx}(x^2)=3(2x)=6x.onumber]$ Since $(f(x)=x^2)$ has derivative of (g(x)) is 3 times the derivative of (g(x)) is 3 times the derivative of $(f(x)=x^2)$ has derivative of $(f(x)=x^2)$ has derivative of (g(x)) is 3 times the derivative of $(f(x)=x^2)$ has derivative of $(f(x)=x^2)$ has derivative of (g(x)) is 3 times the derivative of $(f(x)=x^2)$ has derivative $(f(x)=x^2)$ Figure $(PageIndex{1})$: The derivative of (g(x)) is 3 times the derivative of (f(x)). Example $(PageIndex{5})$: Applying Basic Derivative of (f(x)). rule for differentiating powers. To better understand the sequence in which the differentiation rules are applied, we use Leibniz notation throughout the solution: \(\begin{align*} f(x)&=\dfrac{d}{dx}(2x^5)+\dfrac{d}{dx}(7) & & \text{Apply the sum rule.} \\[4pt] &=2\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{d}{dx}(x^5)+\dfrac{dx}(x^5)+\dfrac{dx}(x^5)+\dfrac{dx}(x^5)+\dfrac{dx}(x^5)+\dfra & $\text{Apply the constant multiple rule.}\[4pt] &= 2(5x^4) + 0 & (\real add the constant rule.}\[4pt] &= 10x^4 & Simplify. \end{align*}) Exercise ((\real add the constant rule.)) Example ((\real add the constant rule.)) Find the derivative of ((f(x)=2x^3-6x^2+3.)) Hint Use the preceding example as a guide. Answer ((f(x)=6x^2-12x.)) Example ((\real add the constant rule.)) ((f(x)=2x^3-6x^2+3.)) Hint Use the preceding example as a guide. Answer ((f(x)=6x^2-12x.)) Example ((\real add the constant rule.)) ((f(x)=2x^3-6x^2+3.)) Hint Use the preceding example as a guide. Answer ((f(x)=6x^2-12x.)) Example ((\real add the constant rule.)) ((f(x)=2x^3-6x^2+3.)) Hint Use the preceding example as a guide. Answer ((f(x)=6x^2-12x.)) Example ((\real add the constant rule.)) ((f(x)=2x^3-6x^2+3.)) Hint Use the preceding example as a guide. Answer ((f(x)=6x^2-12x.)) Example ((\real add the constant rule.)) ((f(x)=2x^3-6x^2+3.)) Hint Use the preceding example as a guide. Answer ((f(x)=6x^2-12x.)) Example ((\real add the constant rule.)) ((f(x)=2x^3-6x^2+3.)) Hint Use the preceding example add the constant rule.) ((f(x)=2x^3-6x^2+3.)) Hint Use the preceding example ((\real add the constant rule.)) ((f(x)=2x^3-6x^2+3.)) Hint Use the preceding example ((\real add the constant rule.)) ((f(x)=2x^3-6x^2+3.)) Hint Use the preceding example ((\real add the constant rule.)) ((f(x)=2x^3-6x^2+3.)) Hint Use the preceding example ((\real add the constant rule.)) ((f(x)=2x^3-6x^2+3.)) Hint Use the preceding example ((\real add the constant rule.)) ((f(x)=2x^3-6x^2+3.)) Hint Use the preceding example ((\real add the constant rule.)) ((f(x)=2x^3-6x^2+3.)) Hint Use the preceding example ((\real add the constant rule.)) ((f(x)=2x^3-6x^2+3.)) Hint Use the preceding example ((\real add the constant rule.)) ((f(x)=2x^3-6x^2+3.)) ($ the Equation of a Tangent Line Find the equation of the line tangent to the graph of $(f(x)=x^2-4x+6)$ at (x=1) Solution To find the point, compute $(f(1)=1^2-4(1)+6=3)$. find (f(x)). Using the definition of a derivative, we have [f(x)=2x-4 onumber] so the slope of the tangent line is (y-3=-2(x-1)). Using the equation of the line in slope-intercept form, we obtain (y=-2x+5). Using the point-slope formula, we see that the equation of the tangent line is (f(1)=-2). Using the point-slope formula, we see that the equation of the tangent line is (f(1)=-2). Using the point-slope formula, we see that the equation of the tangent line is (f(1)=-2). Using the point-slope formula, we see that the equation of the tangent line is (f(1)=-2). Using the point-slope formula, we see that the equation of the tangent line is (f(1)=-2). Using the point-slope formula, we see that the equation of the tangent line is (f(1)=-2). Using the point-slope formula, we see that the equation of the tangent line is (f(1)=-2). Using the point-slope formula, we see that the equation of the tangent line is (f(1)=-2). the equation of the line tangent to the graph of \(f(x)=3x^2-11\) at \(x=2\). Use the point-slope form. Hint Use the preceding example as a guide. Answer \(y=12x-23\) Now that we have examined the basic rules, we can begin looking at some of the more advanced rules. The first one examines the derivative of two functions. Although it might be tempting to assume that the derivative of the product is the product of the derivatives, similar to the sum and difference rules, the product rule does not follow this pattern. To see why we cannot use this pattern. Rule Let (f(x)) and (g(x)) be differentiable functions. Then $[(dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{d}{dx}(f(x)g(x))=(f(x)g(x)+dfrac{dx}(f(x)g(x))=(f(x)g(x)+dfrac{dx}(f(x)g(x))=(f(x)g(x)+dfrac{dx}(f(x)g(x))=(f(x)g(x)+dfrac{dx}(f(x)g(x))=(f(x)g(x)+dfrac{dx}(f(x)g(x))=(f(x)g(x)+dfrac{dx}(f(x)g(x))=(f(x)g(x)+dfrac{dx}(f(x)g(x))=(f(x)g(x)+dfrac{dx}(f(x)g(x))=(f(x)g(x)+dfrac{dx}(f(x)g(x))=(f(x)g(x)+dfrac{dx}(f(x)g(x))=(f(x)g$ the derivative of the second function. Proof We begin by assuming that (g(x)) are differentiable functions. At a key point in this proof we need to use the fact that, since (g(x)) is continuous. In particular, we use the fact that since (g(x)) is differentiable functions. At a key point in this proof we need to use the fact that, since (g(x)) is differentiable functions. At a key point in this proof we need to use the fact that since (g(x)) is differentiable functions. At a key point in this proof we need to use the fact that since (g(x)) is differentiable functions. At a key point in this proof we need to use the fact that since (g(x)) is differentiable functions. At a key point in this proof we need to use the fact that since (g(x)) is differentiable functions. At a key point in this proof we need to use the fact that since (g(x)) is differentiable functions. At a key point in this proof we need to use the fact that since (g(x)) is differentiable functions. 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At a key point in this proof we By applying the limit definition of the derivative to (p(x)=f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)sum law for limits, the derivative becomes $[p'(x)=\lim_{h\to 0}dfrac{f(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)g(x+h)-f(x)$ $-f(x) \{h\} (x) = f'(x)g(x) + (g(x)), and g(x)), and g(x)), and g(x)), and g(x)), and g(x)), and g(x), and g(x)), and g(x$ Rule to Constant Functions For (p(x)=f(x)g(x)), use the product rule to find (p'(2)) if (f(2)=3,); f'(2)=-4,; g(2)=1), and (g'(2)=6). Solution Since (p(x)=f(x)g(x)+g'(x)f(x),) and hence (p'(2)=f'(2)g(2)+g'(2)f(2)=(-4)(1)+(6)(3)=14.) Example (P(x)=f(x)g(x)), (p'(x)=f'(x)g(x)+g'(x)f(x),) and hence (p'(2)=f'(2)g(2)+g'(2)f(2)=(-4)(1)+(6)(3)=14.) Example (P(x)=f(x)g(x), (p'(x)=f'(x)g(x)+g'(x)f(x), (p'(x)=f'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(x)g(x)+g'(find (p'(x)) by applying the product rule. Check the result by first finding the product and then differentiating. Solution If we set $(f(x)=3x^3-5x)$, then $(p'(x)=f'(x)g(x)+g'(x)f(x)=(2x)(3x^3-5x)+(9x^2-5)(x^2+2))$. Thus, $(p'(x)=f'(x)g(x)+g'(x)f(x)=(2x)(3x^3-5x)+(9x^2-5)(x^2+2))$. that $(p(x)=3x^5+x^3-10x)$ and use the preceding example as a guide. Answer $(p'(x)=15x^4+3x^2-10.)$ Hint Set $(f(x)=2x^5)$ and $(g(x)=4x^2+x)$ and use the preceding example as a guide. Answer $(p'(x)=10x^4(4x^2+x)+(8x+1)(2x^5)=56x^6+12x^5)$ Having developed and practiced the product rule, we now consider differentiating quotients of functions. As we see in the following theorem, the derivative of the function in the denominator times the function in the numerator, all divided by the square of the function in the denominator. In order to better grasp why we cannot simply take the quotient Rule Let (f(x)) and $d^{dx}(x^3) = \frac{1}{3x^2} = \frac{1}{3x^2}$. (g(x)) be differentiable functions. Then $[dfrac{d}{dx}(f(x))\cdot g(x)-dfrac{d}{dx}(f(x))\cdot g(x)-dfrac{d}{dx}(g(x))\cdot f(x)}{big(g(x)big)^2}$. In the proof of the product rule is very similar to the proof of the product $[q(x)-dfrac{d}{dx}(f(x))\cdot g(x)-dfrac{d}{dx}(g(x))\cdot f(x)]$ rule, so it is omitted here. Instead, we apply this new rule for finding derivatives in the next example. Example (\PageIndex{9}\): Applying the Quotient rule to find the derivative of $(q(x)=dfrac{5x^2}{4x+3}.)$ Solution Let $(f(x)=5x^2)$ and (g'(x)=4x+3). Thus, (f'(x)=10x) and (g'(x)=4x+3). we have $\left[q'(x)=\frac{f(x)g(x)-g'(x)f(x)}{(g(x))^2}=\frac{10x(4x+3)-4(5x^2)}{(4x+3)^2}\right]$. Answer Apply the quotient rule with $\left(f(x)=\frac{10x(4x+3)-4(5x^2)}{(4x+3)^2}\right)$. Answer $\left(f'(x)=\frac{10x(4x+3)-4(5x^2)}{(4x+3)^2}\right)$. $dfrac{13}{(4x-3)^2}.)$ It is now possible to use the quotient rule to extend the power rule to find derivatives of functions of the form (x^k) where (k) is a negative integer. Extended Power Rule If (k) is a negative integer. The power Rule If (k) is a negative integer. integer with (k=-n). Since for each positive integer $(n),(x^{n-1})$, we may now apply the quotient rule by setting (f(x)=1) and $(g(x)=x^n)$. In this case, (f'(x)=0) and $(g'(x)=x^n)^2$.onumber] Simplifying, we see that $(begin{align*} dfrac{d}{d}$. $(x^{-n}) = \frac{1}{x^2n} (1-1)-2n} (1$ power rule with (k=-4), we obtain $[\frac{1}{x^2})$. Solution It may seem tempting to use the quotient rule to find $(f(x)=\frac{1}{x^2})$. Solution It may seem tempting to use the quotient rule to find this derivative, and it would certainly not be incorrect to do so. However, it is far easier to differentiate this function by first rewriting it as $f(x)=6x^{-2}$, $f(x)=6x^{-2}$, f(x)=6x $dx = 12x^{-3} \& \det x^{-3} \$ $(g(x)=\langle f(x^{7})=x^{(-7)})$. Use the extended power rule with (k=-7). Answer $(g'(x)=-7x^{(-8)})$. As we have seen throughout the examples in this section, it seldom happens that we are called on to apply just one differentiation rule to find the derivative of a given function. At this point, by combining the differentiation rules, we may find the derivatives of any polynomial or rational function. Later on we will encounter more complex combinations of differentiation Rules For \ $(k(x)=3h(x)+x^2g(x))$, find (k'(x)). Solution: Finding this derivative requires the sum rule, the constant multiple rule, and the product rule. $(k'(x)=dfrac{d}{dx})$ $(g(x))x^{2}(x)(x))$ Apply the constant multiple rule to differentiate (3h(x)) and the product rule to differentiate $(x^{2}g(x))$. $(=3h'(x)+2xg(x)+g'(x)x^{2})$ Example $((x^{2}g(x)))$, $(=3h'(x)+2xg(x)+g'(x)x^{2})$ Example $((x^{2}g(x)))$, $(=3h'(x)+2xg(x)+g'(x)x^{2})$ Example $((x^{2}g(x)))$. $(=3h'(x)+2xg(x)+g'(x)x^{2})$ Example $((x^{2}g(x)))$, $(=3h'(x)+2xg(x)+g'(x)x^{2})$ Example $((x^{2}g(x)))$, $(=3h'(x)+2xg(x)+g'(x)x^{2})$ Example $((x^{2}g(x)))$. $(=3h'(x)+2xg(x)+g'(x)x^{2})$ Example $((x^{2}g(x)))$. $(=3h'(x)+2xg(x)+g'(x)x^{2})$ Example $((x^{2}g(x)))$ and the product rule to differentiate $(x^{2}g(x)))$. $(=3h'(x)+2xg(x)+g'(x)x^{2})$ Example $((x^{2}g(x)))$ and the product rule to differentiate $(x^{2}g(x))$. $(=3h'(x)+2xg(x)+g'(x)x^{2})$ Example $((x^{2}g(x)))$ and the product rule to differentiate $(x^{2}g(x))$. $(=3h'(x)+2xg(x)+g'(x)x^{2})$ Example $((x^{2}g(x)))$ and the product rule to differentiate $(x^{2}g(x))$. $(=3h'(x)+2xg(x)+g'(x)x^{2})$ Example $((x^{2}g(x)))$ and the product rule to differentiate $(x^{2}g(x))$ and $(x) = \frac{d}{dx}(2x^3k(x))(3x+2) - \frac{d}{dx}(2x^3k(x))(3x+2) - \frac{d}{dx}(3x+2)(2x^3k(x))}{(3x+2)^2} & \frac{d}{dx}(3x+2)^2 & \frac{d}{dx}$ $= \frac{-6x^3k(x)+18x^3k(x)+12x^2k(x)+6x^4k'(x)+4x^3k'(x)}{(3f(x)-2g(x))}$ Exercise $(\frac{15}{)}$ Exercise $(\frac{15}{)}$ Exercise $(\frac{15}{3})$ Exercise $(\frac{15}{$ Tangent Determine the values of (x) for which $(f(x)=3x^2-14x+8=(3x-2)(x-4))$, we must solve $((x_1-2)(x-4)=0)$. Thus we see that the function has horizontal tangent lines at $(x_1-2)(x-4)=0$. $(x = \frac{2}{3})$ and (x = 4). Example (x = 2/3) and (x = 4). Solution Since the initial velocity is (v(0)=s'(0),) begin by finding (s'(t)) by applying the quotient rule: $(s'(t)=dfrac{1-t^2}{(t^2+1)^2})$. After evaluating, we see that (v(0)=1,) Exercise $(t^2+1)^2$. After evaluating, we see that (v(0)=1,) Exercise $(t^2+1)^2$. line parallel to the line \(y=2x+3.) Hint Solve \(f(x)=2)). Answer \(\dfrac{5}{8}) Formula One car races can be very exciting to watch and attract a lot of spectators. Formula One track designers have to ensure sufficient grandstand space is available around the track to accommodate these viewers. However, car racing can be dangerous, and safety considerations are paramount. The grandstands must be placed where spectators will not be in danger should a driver lose control of a car (Figure \(\PageIndex{3}\)). Figure \(\PageIndex{3}\): The grandstand next to a straightaway of the Circuit de Barcelona-Catalunya race track, located where the spectators are not in danger. Safety is especially a concern on turns. If a driver does not slow down enough before entering the turn, the car may slide off the racetrack. Normally, this just results in a wider turn, which slows the driver down. But if the driver down. But if the driver down. But if the driver down enough before entering the turn, which slows the driver down. But if th Suppose you are designing a new Formula One track. One section of the first straightaway and around a portion of the first curve. The plans call for the front corner of the grandstand to be located at the point (\(-1.9,2.8\)). We want to determine whether this location puts the spectators in danger if a driver loses control of the car. Figure \(PageIndex{4}): (a) One section of the car. Figure \((-1.9,2.8\)). Physicists have determined that drivers are most likely to lose control of their cars as they are coming into a turn, at the point where the slope of the tangent line to the curve at this point. To determine whether the spectators are in danger in this scenario, find the \((x,y)) coordinate of the point where the slope of the tangent line is 1. Find the vertex as they are coming into a turn, at the point where the slope of the tangent line is 1. Find the vertex as they are coming into a turn, at the point where the slope of the tangent line is 1. Find the vertex as they are coming into a turn, at the point where the slope of the tangent line is 1. Find the vertex as they are coming into a turn, at the point where the slope of the tangent line is 1. where the tangent line crosses the line \(y=2.8\). Is this point safely to the right of the grandstand? Or are the spectators in danger? What if a driver loses control at the point (\(-2.5, 0.625\)). What is the slope of the tangent line at this point? If a driver loses control as described in part 4, are the spectators safe? Should you proceed with the current design for the grandstand, or should the grandstand, or should the grandstand, or should the grandstands be moved? Key Concepts The derivative of a constant function is zero. The derivative of a constant function is zero. by 1. The derivative of a constant (c) multiplied by the derivative of (f) and the derivative of (f) and a function (f) and a function (f) is the same as the same difference of the derivative of \(g\). The derivative of the second function times the second function times the second function minus the derivative of the second function, all divided by the second function. We used the limit definition of the derivatives without resorting to the derivative to develop formulas that allow us to find derivative. the derivative of a constant (c) multiplied by the derivative of a constant function (f) is the same as the constant function (f) and a function (g) is the same as the difference of the derivative of $(f_{x}) = f(x) - g(x)$ power rule the derivative of $(g_{x}) = f(x) - g(x)$ power rule the derivative of $(g_{x}) = f(x) - g(x)$ power rule the derivative of $(g_{x}) = f(x) - g(x)$ $dx = \frac{1}{y} = \frac{1}{y}$ the first function times the second function, all divided by the second function, $(f(x)=\frac{d}{dx})=\frac{d}{dx}(g(x))$ sum rule the derivative of the second function $(f(x)=\frac{d}{dx})$ the derivative of (f) and the derivative of (g): $(dfrac{d}{dx}) = f(x) + g'(x)$ Contributors and Attributions Gilbert Strang (MIT) and Edwin "Jed" Herman (Harvey Mudd) with many contributing authors. 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