Differentiation of division of two functions

What if I told you that there was a really easy way to remember the quotient rule for derivatives? Jenn, Founder Calcworkshop®, 15+ Years Experience (Licensed & Certified Teacher) Anything to make calculus easier right? Well then, let’s go! What Is The Quotient Rule The quotient rule is a method for differentiating problems where one function is divided by another. The premise is as follows: If two differentiable functions, f(x) and g(x), exist, then their quotient is also differentiable (i.e., the derivative of the quotient of these two functions also exists). Discovered by Gottfried Wilhelm Leibniz and Isaac Newton, the formula goes like this: Quotient Rule Formula Fun Mnemonic – Hi Dee Ho Now, I have to admit that this formula looks a bit intimidating at first, but there’s an easy pattern to help us remember it and a fun saying that makes memorizing the steps super easy! The trick is to always start with the bottom function and end with the bottom function squared. I like to call this function “Ho” because it’s “low,” and that means the top function is called “Hi” because it’s “high up.” So, the saying goes something like this — Ho-Dee-Hi minus Hi-Dee-Ho over Ho-Ho! Quotient Rule Mnemonic See, isn’t it fun to say? The “Dee” represents the derivative of the function, as the quotient rule is formally read as the bottom times the derivative of the top, minus the top times the derivative of the bottom, all divided by the bottom squared. But let’s be honest, it’s much more fun to say “HoHo” than “bottom squared,” am I right? Quotient Rule Examples As always, it’s best to see how this formula works in practice, so let’s look at an example where we will use the quotient rule to differentiate the function below. Ex) Derivative Of A Fraction Quotient Rule Derivative Ex) Common Mistake And just as we saw with the product rule, I want to caution you and help you to avoid a common mistake. The derivative of a quotient is not equal to the quotient of the derivatives, as the example below nicely demonstrates. The Derivative Quotient Does NOT Equal The Quotient Derivative What is so interesting about this derivative rule is how closely it relates to our understanding of the product rule, except for a minus instead of a plus. But there is a small warning. While the quotient rule is super easy to remember, thanks to our fun saying, as Paul’s Online Notes so accurately states, the numerator of the quotient rule is very similar to the product rule. So please take your time and check your signs! Ex) Instantaneous Rate Of Change Let’s look at another example so you can get more comfortable with this new differentiation rule. Evaluate The Derivative Of The Function At The Given Point So, we just found the instantaneous rate of change for f(x) when x = 1, all thanks to our new rule! Cool, huh? Together we will work through several examples in detail, color coding, and chanting our fun Ho-dee-Hi jingle along the way, so you will quickly master this fantastic rule. And we will look at how to quotient rule helps us find the rate of change, or instantaneous velocity, of one function divided by another. Let’s get to it! Video Tutorial w/ Full Lesson & Detailed Examples (Video) Get access to all the courses and over 150 HD videos with your subscription Monthly, Half-Yearly, and Yearly Plans Available Get My Subscription Now Not yet ready to subscribe? Take Calcworkshop for a spin with our FREE limits course In calculus, the quotient rule is used to find the derivative of a function which can be expressed as a ratio of two differentiable functions. In other words, the quotient rule allows us to differentiate functions which are in fraction form. Say for example we had two functions: f(x) = x2 andg(x) = x Now say we wanted to find the derivative of One approach to find the derivative would be to simplify the function and then differentiate it. So the derivative of the fraction f(x)/g(x) is just 1. Now, out of interest, let’s calculate f’(x)/g’(x) (by differentiating the numerator and then the denominator) So we can see that (f(x)/g(x))’ = 1, and that is not equal to (f’(x)/g’(x)) = 2x The derivative of a quotient is not equal to the quotient of derivatives. To differentiate a quotient, you cannot just take the derivative of the numerator and divide it by the derivative of the denominator. Instead, we need to use the quotient rule to find the derivative of a quotient (in a similar way that the product rule needs to be used to find the derivative of a product). The Quotient Rule Formula For Differentiation If two functions f(x) and g(x) are differentiable (i.e. the derivatives of f(x) and g(x) exist), then their quotient (f(x)/g(x)) is differentiable, and the derivative can be found as follows: The formula for the quotient rule in words The Quotient Rule states that the derivative of a quotient is equal to the denominator times the derivative of the numerator minus the numerator times the derivative of the denominator, all divided by the square of the denominator. Two tricks to remember the quotient rule Similar to the product rule, you need to find the derivative of f(x) and then multiply it with g(x) and then find the derivative of g(x) and multiply it by f(x). However, because the terms are subtracted, the order matters, and there is also a division which needs to be performed. This makes the quotient rule a little more difficult to memorize compared to the product rule. Here are two methods which may help you remember the quotient rule: 1. You always start with the bottom function (denominator) and end with the bottom function squared. And if that doesn’t work… 2. Here is a jingle which you can use: Lo d hi minus hi d low, all over the square of what’s below. Here the lo, refers to the denominator, hi refers to the numerator and d refers to “the derivative of”. It’s really silly, but that’s why it works! When to use the quotient rule In calculus, the quotient rule can be applied when the function you want to differentiate consists of a quotient (or fraction), and the numerator and denominator of the quotient are both differentiable functions of their own. For example the function f(x) = x/x2 can be differentiated using the quotient rule for derivatives because: The numerator (x) is differentiable – its derivative is 1The denominator (x2) is differentiable – it’s derivative is x (Of course you could also first simplify the function to 1/x and then differentiate it and get the same result) Examples Using The Quotient Rule The best way to see how the quotient rule works is by looking at some examples. Using the quotient rule For the first example, let’s find the derivative of Let’s call the numerator f(x), so f(x) = x2 + 1Let’s call the denominator g(x), so g(x) = x3 This means that:f’(x) = 2xg’(x) = 3x2 We can now apply the quotient rule to find the derivative of f(x)/g(x). The formula for the quotient rule is Now we can plug f(x), f'(x), g(x) and g'(x) into the formula And then we just simplify using factorization and the rules of exponents to find the final answer Using the quotient rule with trig functions As another example, we can use the quotient rule to find the derivative of tan(x) First recall that tan(x) can be expressed as sin(x) divided by cos(x) Let’s call the numerator f(x), so f(x) = sin(x)Let’s call the denominator g(x), so g(x) = cos(x) This means that:f'(x) = cos(x)g'(x) = -sin(x) We can now apply the quotient rule to find the derivative of f(x)/g(x). The formula for the quotient rule is Now we can plug f(x), f'(x), g(x) and g'(x) into the formula: Then simplify remembering the trig identity sin2(x) + cos2(x) = 1 and recalling that 1/cos(x) is equal to sec(x). Using the quotient rule, the derivative of tan(x) is equal to sec2(x) Product and Quotient Rule Proof of the Quotient Rule There are a number of ways to prove the quotient rule. Here we will look at proving the quotient rule using: First principles – the derivative definition and properties of limits.Implicit differentiation and the product ruleThe product and chain rulesNatural logarithms and the chain rule Proving the quotient rule using first principles Let F(x) = f(x)/g(x) The definition for the derivative of F(x) is Then we put F(x+h) and F(x) in This can be rewritten by finding the lowest common denominator in the numerator (g(x+h).g(x)) and taking the 1/h out. The next step is needed to make life easier a little later on. What we do is we add 0 to the numerator, which does not change the value at all. But instead of adding a straight 0, we add it in the form -f(x)g(x) + f(x)g(x) Now we swap the two denominators around – we are allowed to do this since we are multiplying fractions. Next we use one of the properties of limits which says that the limit of a product of two functions is equal to the product of their limits. We then also use the fact that the limit as h approaches zero of g(x+h) is simply g(x) Now we split the large fraction up, and take out a common g(x) from the first part and a common -f(x) from the second part. Another property of limits is that the limit of a sum is equal to the sum of the limits. So we can take the limit of each fraction separately. And then we can take the limit of each part of the products Now you may notice that two of the above limits look familiar – they are the exact definitions for f'(x) and g'(x). Lastly, the limit as h approaches 0 of g(x) and f(x) remain g(x) and f(x) (since they are not dependent on h). And then all that is left to do is to rewrite everything in the simplest form. And there we have it – the formula for the quotient rule. Proving the quotient rule using implicit differentiation and the product rule Let y = f(x)/g(x), then we want to find y’. First let’s rearrange the equation: Now we can take the derivative on the left and right hand sides of the equation (by using the product rule on the right hand side) Next, substitute y = f(x)/g(x) into the above equation We ultimately want to find the derivative of y (y’) so now we solve for y’ and find the formula for the quotient rule. Proving the quotient rule using the product and chain rules Let F(x) = f(x)/g(x). We can rewrite this quotient as a product We can find F'(x) by using the product rule To find (g(x)-1)’ we can apply the chain rule. The chain rule says we first take the derivative of g(x)-1 in terms of g(x) (= (-1)g(x)-2) and then multiply that by the derivative of g(x) in terms of x (= g'(x)). Next we bring the negative exponents to the denominator and then simplify by finding the greatest common denominator. We are left with the formula for the quotient rule. Proving the quotient rule using ln and the chain rule Let y = f(x)/g(x) To start, take the natural logarithm (ln) on both sides Next apply the properties of logs to split the right hand side’s quotient to a difference of logs Now take the derivative on both sides. Recall that the derivative of ln(x) with respect to x is 1/x. In a similar way, the derivative of ln(y) with respect to y is 1/y, the derivative of ln(f(x)) with respect to f(x) is 1/f(x) and the derivative of ln(g(x)) with respect to g(x) is 1/g(x). To find the derivatives of each of the terms with respect to x, we can apply the chain rule by first differentiating with respect to the inner function of ln, and then multiplying that by the derivative of the inner function. Lastly, isolate y’, substitute y = f(x)/g(x) in, and then simplify to solve for y’ The quotient rule in terms of u and v With regards to notation, the quotient rule is sometimes easier to express if you use u and v to represent the numerator and denominator respectively. This can be more compact than using the function notation f(x) and g(x). Using the variable u for the numerator, and v for the denominator, the quotient rule for finding the derivative of the function u/v can be expressed as: DerivativeIt lets you quickly look up derivatives, but also shows you the full calculations for finding derivatives of trigonometric, exponential and natural logarithmic functions Learning Objectives State the constant, constant multiple, and power rules. Apply the sum and difference rules to combine derivatives. Use the product rule for finding the derivative of a product of functions. Use the quotient rule for finding the derivative of a quotient of functions. Extend the power rule to functions with negative exponents. Combine the differentiation rules to find the derivative of a polynomial or rational function. Finding derivatives of functions by using the definition of the derivative can be a lengthy and, for certain functions, a rather challenging process. For example, previously we found that \[\dfrac{d}{dx}\left(\sqrt{x}\right)=\dfrac{1}{2\sqrt{x}} onumber\] by using a process that involved multiplying an expression by a conjugate prior to evaluating a limit. The process that we could use to evaluate \(\dfrac{d}{dx}\left(\sqrt[3]{x}\right)\) using the definition, while similar, is more complicated. In this section, we develop rules for finding derivatives that allow us to bypass this process. We begin with the basics. The functions \(f(x)=c\) and \(g(x)=x^n\) where \(n\) is a positive integer are the building blocks from which all polynomials and rational functions are constructed. To find derivatives of polynomials and rational functions efficiently without resorting to the limit definition of the derivative, we must first develop formulas for differentiating these basic functions. We first apply the limit definition of the derivative to find the derivative of the constant function, \(f(x)=c\). For this function, both \(f(x)=c\) and \(f(x+h)=c\), so we obtain the following result: \[\begin{align*} f′(x) &=\lim_{h→0} \dfrac{f(x+h)−f(x)}{h} \\[4pt] &=\lim_{h→0}\dfrac{c−c}{h} \\[4pt] &=\lim_{h→0}\dfrac{0}{h} \\[4pt] &=\lim_{h→0}0=0. \end{align*}\] The rule for differentiating constant functions is called the constant rule. It states that the derivative of a constant function is zero; that is, since a constant function is a horizontal line, the slope, or the rate of change, of a constant function is \(0\). We restate this rule in the following theorem. The Constant Rule Let \(c\) be a constant. If \(f(x)=c\), then \(f′(c)=0.\) Alternatively, we may express this rule as \[\dfrac{d}{dx}(c)=0.\] Example \(\PageIndex{1}\): Applying the Constant Rule Find the derivative of \(f(x)=8.\) Solution This is just a one-step application of the rule: \(f′(8)=0.\) Exercise \(\PageIndex{1}\) Find the derivative of \(g(x)=−3\). Hint Use the preceding example as a guide Answer 0 We have shown that \[\dfrac{d}{dx}\left(x^2\right)=2x\quad\text{ and }\quad\dfrac{d}{dx}\left(x^{1/2}\right)=\dfrac{1}{2}x^{−1/2}. onumber\] At this point, you might see a pattern beginning to develop for derivatives of the form \(\dfrac{d}{dx}\left(x^n\right)\). We continue our examination of derivative formulas by differentiating power functions of the form \(f(x)=x^n\) where \(n\) is a positive integer. We develop formulas for derivatives of this type of function in stages, beginning with positive integer powers. Before stating and proving the general rule for derivatives of functions of this form, we take a look at a specific case, \(\dfrac{d}{dx}(x^3)\). As we go through this derivation, pay special attention to the portion of the expression in boldface, as the technique used in this case is essentially the same as the technique used to prove the general case. Example \(\PageIndex{2}\): Differentiating \(x^3\) Find \(\dfrac{d}{dx}\left(x^3\right)\). Solution: \(\displaystyle \dfrac{d}{dx}\left(x^3\right)=\lim_{h→0}\dfrac{(x+h)^3−x^3}{h}\) \(\displaystyle =\lim_{h→0}\dfrac{x^3+3x^2h+3xh^2+h^3−x^3}{h}\) Notice that the first term in the expansion of \((x+h)^3\) is \(x^3\) and the second term is \(3x^2h\). All other terms contain powers of \(h\) that are two or greater \(\displaystyle =\lim_{h→0}\dfrac{3x^2h+3xh^2+h^3}{h}\) In this step the \(x^3\) terms have been cancelled, leaving only terms containing \(h\). \(\displaystyle =\lim_{h→0}\dfrac{h(3x^2+3xh+h^2)}{h}\) Factor out the common factor of \(h\). \(\displaystyle =\lim_{h→0}(3x^2+3xh+h^2)\) After cancelling the common factor of \(h\),the only term not containing \(h\) is \(3x^2\). \(=3x^2\) Let \(h\) go to \(0\). Exercise \(\PageIndex{2}\) Find \(\dfrac{d}{dx}\left(x^4\right).\) Hint Use \((x+h)^4=x^4+4x^3h+6x^2h^2+4xh^3+h^4\) and follow the procedure outlined in the preceding example. Answer \(\dfrac{d}{dx}\left(x^4\right) = 4x^3\) As we shall see, the procedure for finding the derivative of the general form \(f(x)=x^n\) is very similar. Although it is often unwise to draw general conclusions from specific examples, we note that when we differentiate \(f(x)=x^3\), the power on \(x\) becomes the coefficient of \(x^2\) in the derivative and the power on \(x\) in the derivative decreases by 1. The following theorem states that the power rule holds for all positive integer powers of \(x\). We will eventually extend this result to negative integer powers. Later, we will see that this rule may also be extended first to rational powers of \(x\) and then to arbitrary powers of \(x\). Be aware, however, that this rule does not apply to functions in which a constant is raised to a variable power, such as \(f(x)=3^x\). The Power Rule Let \(n\) be a positive integer. If \(f(x)=x^n\),then \[f′(x)=nx^{n−1}.\] Alternatively, we may express this rule as \[\dfrac{d}{dx}\left(x^n\right)=nx^{n−1}.\] Proof For \(f(x)=x^n\) where \(n\) is a positive integer, we have \[f′(x)=\lim_{h→0}\dfrac{(x+h)^n−x^n}{h}. onumber\] Since \((x+h)^n=x^n+nx^{n−1}h+\binom{n}{2}x^{n−2}h^2+\binom{n}{3}x^{n−3}h^3+…+nxh^{n−1}+h^n,\) we see that \((x+h)^n−x^n=nx^{n−1}h+\binom{n}{2}x^{n−2}h^2+\binom{n}{3}x^{n−3}h^3+…+nxh^{n−1}+h^n.\) Next, divide both sides by h: \(\dfrac{(x+h)^n−x^n}{h}=\dfrac{nx^{n−1}h+\binom{n}{2}x^{n−2}h^2+\binom{n}{3}x^{n−3}h^3+…+nxh^{n−1}+h^n}{h}.\) Thus, \(\dfrac{(x+h)^n−x^n}{h}=nx^{n−1}+\binom{n}{2}x^{n−2}h+\binom{n}{3}x^{n−3}h^2+…+nxh^{n−2}+h^{n−1}.\) Finally, \[f′(x)=\lim_{h→0}(nx^{n−1}+\binom{n}{2}x^{n−2}h+\binom{n}{3}x^{n−3}h^2+…+nxh^{n−1}+h^n) onumber\] \(=nx^{n−1}.\) □ Example \(\PageIndex{3}\): Applying the Power Rule Find the derivative of the function \(f(x)=x^{10}\) by applying the power rule. Solution Using the power rule with \(n=10\), we obtain \[f′(x)=10x^{10−1}=10x^9. onumber\] Exercise \(\PageIndex{3}\) Find the derivative of \(f(x)=x^7\). Hint Use the power rule with \(n=7.\) Answer \(f′(x)=7x^6\) We find our next differentiation rules by looking at derivatives of sums, differences, and constant multiples of functions. Just as when we work with functions, there are rules that make it easier to find derivatives of functions that we add, subtract, or multiply by a constant. These rules are summarized in the following theorem. Sum, Difference, and Constant Multiple Rules Let \(f(x)\) and \(g(x)\) be differentiable functions and \(k\) be a constant. Then each of the following equations holds. Sum Rule. The derivative of the sum of a function \(f\) and a function \(g\) is the same as the sum of the derivative of \(f\) and the derivative of \(g\). \[\dfrac{d}{dx}\big(f(x)+g(x)\big)=\dfrac{d}{dx}\big(f(x)\big)+\dfrac{d}{dx}\big(g(x)\big);\] that is, \[\text{for }s(x)=f(x)+g(x),\quad s′(x)=f′(x)+g′(x). onumber\] Difference Rule.

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